Revision of Maxwell's Equations

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Abstract. It is well known that the Maxwell equations predict the behavior of the electromagnetic field very well. However, they predict only one wave equation while there are significant differences between the "near" and "far" fields and various anomalies have been observed involving the detection of super luminous signals in experiments with microwaves[1,2,3,4,5,6,7,8,9,10], optical fibers[11,12,13,14] as well as other methods[15,16,17,18]. Here we show that the mathematical Laplace operator, when applied to physics, defines a complete set of vector fields consisting of two potential fields and two fields of force, which form a Helmholtz decomposition of any given vector field **F**. We found that neither in Maxwell's equations nor in fluid dynamics vector theory this result has been recognized, which causes the potential fields to not be uniquely defined. We also found that equivalents for the Maxwell equations can be derived from a superfluid medium model using the Laplace operator and then three types of wave phenomena can be described, including super luminous longitudinal sound-like waves that can explain these anomalies.

This paper contributes to the growing body of work revisiting Maxwell's equations[19,20,21,22,23,24,25,26,27], by deriving all of the fields from a single equation, so the result is known to be mathematically consistent and free of singularities and uniquely defines the potential fields. Unlike Maxwell's equations, which are the result of the entanglement of Faraday's circuit level law with the more fundamental medium arguably creating most of the problems in current theoretical physics, these revisions describe the three different electromagnetic waves observed in practice and so enable a better mathematical representation.

Keywords: Classical Electrodynamics, Superfluid medium, Fluid Dynamics, Theoretical Physics, Vector Calculus.

Introduction

In 1861, James Clerk Maxwell published his paper "On Physical Lines of Force"[28], wherein he theoretically derived a set of twenty equations which accurately described the electromagnetic field insofar as known at that time. He modeled the magnetic field using a molecular vortex model of Michael Faraday's "lines of force" in conjunction with the experimental result of Weber and Kohlrausch, who determined in 1855 that there was a quantity related to electricity and magnetism, the ratio of the absolute electromagnetic unit of charge to the absolute electrostatic unit of charge, and determined that it should have units of velocity. In an experiment, which involved charging and discharging a Leyden jar and measuring the magnetic force from the discharge current, they found a value 3.107 x 10^8 m/s, remarkably close to the speed of light.

In 1884, Oliver Heaviside, concurrently with similar work by Josiah Willard Gibbs and Heinrich Hertz, grouped Maxwell's twenty equations together into a set of only four, via vector notation. This group of four equations was known variously as the Hertz-Heaviside equations and the Maxwell-Hertz equations but are now universally known as Maxwell's equations.

The Maxwell equations predict the existence of just one type of electromagnetic wave, even though it is now known that at least two electromagnetic wave phenomena exist, namely the "near" and the "far" fields. The "near" field has been shown to be a non-radiating surface wave that is guidable along a completely unshielded single conductor[29] and can be applied for wideband low loss communication systems. The Maxwell equations have not been revised to incorporate this new knowledge.

Given the above, the following questions should be asked:

- What is charge?
- Why is it a property of certain particles?

As long as we insist that charge is an elemental quantity that is a property of certain particles, we cannot answer these questions. Also, when the wave particle duality principle is considered in relation to what is considered to be the cause for electromagnetic radiation, charged particles, in Maxwell's equations electromagnetic radiation is essentially considered to be caused by (quanta of) electromagnetic radiation, an obvious case of circular logic which is not desirable.

In the area of vector calculus, Helmholtz's theorem, also known as the fundamental theorem of vector calculus, states that any sufficiently smooth, rapidly decaying vector field in three dimensions can be resolved into the sum of an irrotational (curl-free) vector field and a solenoidal (divergence-free) vector field; this is known as the Helmholtz decomposition. A terminology often used in physics refers to the curl-free component of a vector field as the longitudinal component and the divergence-free component as the transverse component. This theorem is also of great importance in electromagnetic (EM) and microwave engineering, especially for solving the low-frequency breakdown issues caused by the decoupling of electric and magnetic fields.[30] Further, a vector field can be uniquely specified by a prescribed divergence and curl and it can be shown that the Helmholtz theorem holds for arbitrary vector fields, both static and time-dependent[31].

In potential theory, the study of harmonic functions, the Laplace equation is very important, amongst other with regards to consideration of the symmetries of the Laplace equation. The symmetries of the n-dimensional Laplace equation are exactly the conformal symmetries of the n-dimensional Euclidean space, which has several implications. One can systematically obtain the solutions of the Laplace equation which arise from separation of variables such as spherical harmonic solutions and Fourier series. By taking linear superpositions of these solutions, one can produce large classes of harmonic functions which can be shown to be dense in the space of all harmonic functions under suitable topologies.

The Laplace equation as well as the more general Poisson equation are 2nd order differential equations, in both of which the Laplacian represents the flux density of the gradient flow of a function. In one dimension, the Laplacian simply is $\partial^2/\partial x^2$, representing the curvature of a given function *f*. For scalar functions in 3D, the Laplacian is a common generalization of the second derivative and is the differential operator defined by:

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} \tag{1}$$

$$\text{LaPlace}(f) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

The Laplacian of a scalar function is equal to the divergence of the gradient and the trace of the Hessian matrix. The vector Laplacian is a further generalization in three dimensions and defines the second order spatial derivative of any given vector field $\boldsymbol{F}$, the "3D curvature" if you will, and is given by the identity:

$$\nabla^2 \boldsymbol{F} = \nabla(\nabla \cdot \boldsymbol{F}) - \nabla \times (\nabla \times \boldsymbol{F}) \tag{2}$$

$$\text{LaPlace}(\boldsymbol{F}) = \textit{grad}\ \text{div}\ \boldsymbol{F} - \textit{curl}\ \textit{curl}\ \boldsymbol{F}$$

Whereas the scalar Laplacian applies to a scalar field and returns a scalar quantity, the vector Laplacian applies to a vector field, returning a vector quantity. When computed in orthonormal Cartesian coordinates, the returned vector field is equal to the vector field of the scalar Laplacian applied to each vector component.

Methods

The terms in the definition for the vector Laplacian can be negated:

$$-\nabla^2 \boldsymbol{F} = -\nabla(\nabla \cdot \boldsymbol{F}) + \nabla \times (\nabla \times \boldsymbol{F}) \tag{3}$$

$$-\text{LaPlace}(\boldsymbol{F}) = -\textit{grad}\ \text{div}\ \boldsymbol{F} + \textit{curl}\ \textit{curl}\ \boldsymbol{F}$$

and then the terms in this identity can be written out to define a vector field for each of these terms:

$$\begin{aligned} \boldsymbol{A} &= \nabla \times \boldsymbol{F} \\ \varphi &= \nabla \cdot \boldsymbol{F} \\ \boldsymbol{B} &= \nabla \times \boldsymbol{A} = \nabla \times (\nabla \times \boldsymbol{F}) \\ \boldsymbol{E} &= -\nabla \varphi = -\nabla(\nabla \cdot \boldsymbol{F}) \end{aligned} \tag{4}$$

$$\begin{aligned} \boldsymbol{A} &= \textit{curl}\ \boldsymbol{F} \\ \varphi &= \text{div}\ \boldsymbol{F} \\ \boldsymbol{B} &= \textit{curl}\ \boldsymbol{A} = \textit{curl}\ \textit{curl}\ \boldsymbol{F} \\ \boldsymbol{E} &= -\textit{grad}\ \varphi = -\textit{grad}\ \text{div}\ \boldsymbol{F} \end{aligned}$$

And, since the curl of the gradient of any twice-differentiable scalar field φ is always the zero vector ($\nabla \times (\nabla \varphi) = 0$) [curl grad φ = 0], and the divergence of the curl of any vector field $\boldsymbol{A}$ is always zero as well ($\nabla(\nabla\times\boldsymbol{A})=0$) [div curl $\boldsymbol{A}$ = 0], we can establish that $\boldsymbol{E}$ is curl-free and $\boldsymbol{B}$ is divergence-free, and we can write:

$$\begin{aligned} \nabla\times\boldsymbol{E} &= 0 \\ \nabla\cdot\boldsymbol{B} &= 0 \end{aligned} \tag{5}$$

$$\begin{aligned} curl\,\boldsymbol{E} &= 0 \\ \mathrm{div}\,\boldsymbol{B} &= 0 \end{aligned}$$

As can be seen from this, the vector Laplacian establishes a Helmholtz decomposition of the vector field $\boldsymbol{F}$ into an irrotational or curl free component $\boldsymbol{E}$ and a divergence free component $\boldsymbol{B}$, along with associated potential fields φ and $\boldsymbol{A}$, all from a single equation c.q. operator.

Thus we have shown that the mathematical definitions for potential fields are hidden within the Laplace operator c.q. the fundamental theorem of vector calculus c.q. the second order spatial derivative, which has tremendous consequences for both the analytical analysis of the electromagnetic field as well as fluid dynamics vector theory. The symmetry between the fields thus defined is fundamental and has been mathematically proven to be correct, so it is vital to maintain this fundamental symmetry in our physics equations.

So far, we have considered the general case, which is valid for any given vector field **F**. In the following, we will use the $_m$ subscript to refer to the electromagnetic domain along Maxwell’s equations, while the $_f$ subscript is used for the fluid dynamics domain.

In Maxwell's equations, the curl of the electric field $\boldsymbol{E}_m$ is defined by:

$$\nabla\times\boldsymbol{E}_m=-\frac{\partial\boldsymbol{B}_m}{\partial t}, \tag{6}$$

$$curl\,\boldsymbol{E}_m=-\frac{\partial\boldsymbol{B}_m}{\partial t},$$

which is obvious not equal to zero for electromagnetic fields varying with time and therefore the dynamic Maxwell equations cannot be second order spatial derivatives of any vector field $\boldsymbol{F}_m$ as defined by the Laplacian.

Herewith, we have shown that no vector field $\boldsymbol{F}_m$ exists for which Maxwell's equations are the second order spatial derivative and therefore Maxwell’s equations do not satisfy the vector Laplace equation. The end result of this is that while the solutions of Laplace’s equation are all possible harmonic wave functions, with Maxwell’s equations there is only one resulting wave equation which defines a “transverse” wave, whereby the $\boldsymbol{E}_m$ and $\boldsymbol{B}_m$ components are always perpendicular with respect to one another. This is also the reason why no separate wave equations can be derived for the “near” and “far” fields.

Furthermore, in Maxwell’s equations, the two potential fields which are used with Helmholtz’s theorem are the electrical potential φ_m and the magnetic vector potential $\boldsymbol{A}_m$, which are defined by the equations[32]:

$$
\begin{aligned}
\boldsymbol{B_m} &= \nabla \times \boldsymbol{A_m} \\
\boldsymbol{E_m} &= -\nabla \varphi_m - \frac{\partial \boldsymbol{A_m}}{\partial t}
\end{aligned} \tag{7}
$$

$$
\begin{aligned}
\boldsymbol{B_m} &= curl\, \boldsymbol{A_m} \\
\boldsymbol{E_m} &= -grad\, \varphi_m - \frac{\partial \boldsymbol{A_m}}{\partial t}
\end{aligned}
$$

where $\boldsymbol{B}_m$ is the magnetic field and $\boldsymbol{E}_m$ is the electric field.

The Helmholtz theorem can also be described as follows. Let **A** be a solenoidal vector field and φ a scalar field on $\mathbf{R}^3$ which are sufficiently smooth and which vanish faster than $1/r^2$ at infinity. Then there exists a vector field **F** such that:

$$
\nabla \boldsymbol{F} = \varphi \text{ and } \nabla \times \boldsymbol{F} = \boldsymbol{A} \tag{8}
$$

$$
\text{div}\, \boldsymbol{F} = \varphi \text{ and } curl\, \boldsymbol{F} = \boldsymbol{A}
$$

and if additionally, the vector field **F** vanishes as $r \to \infty$, then **F** is unique[33].

Now let us consider the units of measurement involved in these fields, whereby the three vector operators used all have a unit of measurement in per meter [/m]. The magnetic field $\mathbf{B}_m$ has a unit of measurement in Tesla [T], which is defined in SI units as [kg/s^2-A]. So, for the magnetic vector potential $\mathbf{A}_m$ we obtain a unit of [kg-m/s^2-A] and for $d\mathbf{A}_m/dt$ we obtain a unit of [kg-m/s^3-A]. The electric field $\mathbf{E}_m$ has a unit of measurement in volt per meter, which is defined in SI units as [kg-m/s^3-A], which matches that for $d\mathbf{A}_m/dt$. So, for the electric scalar potential φ_m we obtain a unit of [kg-m^2/s^3-A].

However, neither the units of measurement for $\mathbf{E}_m$ and $\mathbf{B}_m$ are the same, nor are the units of measurements for φ_m and $\mathbf{A}_m$. This is in contradiction with Helmholtz's theorem, which states that a vector field $\mathbf{F}_m$ exists that should have a unit of measurement equal to that of φ_m and $\mathbf{A}_m$ times meters or that of $\mathbf{E}_m$ and $\mathbf{B}_m$ times meters squared.

Thus, we have shown that Maxwell's equations are in contradiction with Helmholtz's theorem as well, which means that the potential fields defined by Maxwell are mathematically inconsistent and should therefore be revised.

It can be shown[34] that by using the 19th Century's atomic vortex postulate in combination with a superfluid model for the medium, it is possible to construct a single simple integrated model which covers all major branches of physics including kinetic, fluid, gravitation, relativity, electromagnetism, thermal, and quantum theory. With this method, it can also be shown that anomalous observations such as Pioneer's drag and the electron's magnetic moment can be directly accounted for by the model. Furthermore, with this model all units of measurements are defined in terms of just three fundamental units of measurement: mass, length, and time.

It should be noted that there are two distinct levels in this model, with each playing their own role. The first consists of basic media quanta, which forms a superfluid model for the medium itself. The second describes vortices within the fluid, which forms a particle model on top of the medium model. The lower base level is assumed to be an (if not ideal, nearly so) in-viscous superfluid system obeying the defined rules of basic kinetic theory and that is the model this paper is originally based on, which means that the equations presented in this paper do not depend on the higher level Atomic Vortex Hypothesis based model. However, during the course of this work it became clear that viscosity plays a crucial role in our model, which has as consequence that an in-viscous superfluid model is insufficient to describe the behavior of the medium.

Of course, a (viscous) superfluid model can also be described in vector notation and since this model essentially describes a fluid/gas like medium, we can apply continuum mechanics fluid dynamics vector calculus methods to re-derive the Maxwell equations from the basic model. As is common practice in continuum mechanics fluid dynamics vector theory, we can describe its dynamic behavior by working with the medium's flow velocity vector field[35] **v**, with **v** representing the local average bulk flow velocity.

It should be noted that because we use continuum mechanics, the equations presented in this paper are independent on the detailed description of the constituents of the medium itself and that there is a lower limit with respect to scale below which the medium can no longer be considered as a continuum. In that case, the model is no longer applicable, which is a well-known limitation of continuum mechanics. The Knudson number can be used to estimate this limit. However, with our viscous superfluid model, we have left any notion of the constituents the medium itself behind, so at this point we cannot say anything sensible about whether or not such a lower scale limit actually applies.

Within the fluid dynamics domain, a scalar potential field φ_f and a vector potential field $\mathbf{A}_f$ are generally described for an incompressible fluid ($\nabla \mathbf{v}_f = 0$) [div $\boldsymbol{v}_f = 0$] with a flow velocity field $\mathbf{v}_f$ as follows[36] (eq. 17-19):

$$\boldsymbol{v}_f = \nabla \varphi_f + \nabla \times \boldsymbol{A}_f \tag{9}$$

$$\boldsymbol{v}_f = grad\, \varphi_f + curl\, \boldsymbol{A}_f$$

where the velocity potential φ_f is a scalar potential field, satisfying the Laplace equation:

$$\nabla^2 \varphi_f = 0 \tag{10}$$

$$\text{LaPlace}(\varphi_f) = 0$$

and the vorticity potential $\mathbf{A}_f$ is a solenoidal (i.e. $\nabla . \boldsymbol{A}_f = 0$) [div $\boldsymbol{A}_f = 0$] vector potential field satisfying the Poisson equation:

$$\nabla^2 \boldsymbol{A}_f = -\nabla \times (\nabla \times \boldsymbol{A}_f) = -\boldsymbol{\omega}_v, \tag{11}$$

$$\text{LaPlace}(\boldsymbol{A}_f) = -curl\, curl\, \boldsymbol{A}_f = -\boldsymbol{\omega}_v,$$

where $\boldsymbol{\omega}_v = \nabla\times\mathbf{v}_f$ [$\boldsymbol{\omega}_v$= curl $\mathbf{v}_f$] is the velocity vorticity field.

However, with this definition, the potential fields are not uniquely defined and the boundary conditions on φ_f and $\mathbf{A}_f$ depend on the nature of the flow at the boundary of the flow domain and on the topological properties of the flow domain, respectively.

We can can attempt to resolve this problem for the general case of a fluid that is both compressible and rotational by defining a compressible irrotational velocity field $\mathbf{E}_f$ for the scalar potential φ_f and an incompressible solenoidal velocity field $\mathbf{B}_f$ and associated vorticity field $\boldsymbol{\omega}$ for the vector potential $\mathbf{A}_f$ using the Helmholtz decomposition and negating the commonly used definition for the velocity potential φ_f:

$$\boldsymbol{v}_f = -\nabla\varphi_f + \nabla\times\boldsymbol{A}_f = \boldsymbol{E} + \boldsymbol{B} \tag{12}$$

$$\begin{aligned} \boldsymbol{E}_f &= -\nabla\varphi_f \\ \boldsymbol{B}_f &= \nabla\times\boldsymbol{A}_f \\ \boldsymbol{\omega} &= \nabla\times\boldsymbol{B}_f \end{aligned} \tag{13}$$

$$\boldsymbol{v}_f = -grad\,\varphi_f + curl\,\boldsymbol{A}_f = \boldsymbol{E} + \boldsymbol{B}$$

$$\begin{aligned} \boldsymbol{E}_f &= -grad\,\varphi_f \\ \boldsymbol{B}_f &= curl\,\boldsymbol{A}_f \\ \boldsymbol{\omega} &= curl\,\boldsymbol{B}_f \end{aligned}$$

This way, the $\boldsymbol{E}_f$ and $\boldsymbol{B}_f$ fields describe flow velocity fields with a unit of measurement in [m/s] and both the velocity potential and the velocity vorticity potential describe fields with a unit of measurement in meters squared per second [m^2/s]. However, the primary vector field $\boldsymbol{F}_f$ thus has a unit of measurement in [m^3/s], which describes a vector field for a volumetric flow rate or volume velocity. This can be considered as the flow velocity vector field $\mathbf{v}_f$ times a surface S perpendicular to $\mathbf{v}_f$ with a surface area proportional to h^2 square meters [m^2], with h the physical length scale in meters [m]. This results in the zero vector when taking the limit for the length scale h to zero, which is obviously problematic.

So far, we have considered the general mathematical case for the Helmholtz decomposition of any given vector field $\boldsymbol{F}$ as well as its common use in both the electrodynamics and the fluid dynamics domains, whereby we encountered a number of problems. In order to resolve these problems and avoid confusion with the various fields used thus far, let us first introduce a new set of fields along equation (4):

$$
\begin{aligned}
P &= \nabla \cdot \boldsymbol{C} \\
\boldsymbol{\Omega} &= \nabla \times \boldsymbol{C} \\
\boldsymbol{L} &= -\nabla P = -\nabla(\nabla \cdot \boldsymbol{C}) \\
\boldsymbol{R} &= \nabla \times \boldsymbol{\Omega} = \nabla \times (\nabla \times \boldsymbol{C}),
\end{aligned} \tag{14}
$$

$$
\begin{aligned}
P &= \operatorname{div} \boldsymbol{C} \\
\boldsymbol{\Omega} &= curl\, \boldsymbol{C} \\
\boldsymbol{L} &= -grad\, P = -grad(\operatorname{div} \boldsymbol{C}) \\
\boldsymbol{R} &= curl\, \boldsymbol{\Omega} = curl(curl\, \boldsymbol{C}),
\end{aligned}
$$

where $\boldsymbol{C}$ is our primary vector field, P is the scalar potential or pressure, $\boldsymbol{\Omega}$ is the vector potential or angular pressure, $\boldsymbol{L}$ is the longitudinal or translational force density and $\boldsymbol{R}$ is the rotational or angular force density. Hereby, P and $\boldsymbol{\Omega}$ have a unit of measurement in Pascal [Pa] or Newtons per square meter [N/m^2] and $\boldsymbol{L}$ and $\boldsymbol{R}$ are in Newtons per cubic meter [N/m^3]. $\boldsymbol{C}$ is in Newtons per meter [N/m] or kilograms per second squared [kg/s^2], thus representing an as of yet undefined quantity. Further down, we will see that for the medium this unit corresponds to the Ampere, hence the choice for using the symbol $\boldsymbol{C}$.

Let us now consider Newton's second law, expressed in densities or per unit volume:

$$
\boldsymbol{f}_n = \rho \boldsymbol{a} = \rho \frac{d\boldsymbol{v}}{dt} = -\nabla P, \tag{15}
$$

$$
\boldsymbol{f}_n = \rho \boldsymbol{a} = \rho \frac{d\boldsymbol{v}}{dt} = -grad\, P,
$$

with $\boldsymbol{f}_n$ the force density in [N/m^3], ρ the mass density of the fluid, **v** the velocity field, **a** the acceleration field and P the pressure or scalar potential field, defined as the divergence of $\boldsymbol{C}$. Since $\boldsymbol{C}$ should exist and should have a unit of measurement in [kg/s^2] or [N/m], we can define $\boldsymbol{C}$ as follows:

$$
\boldsymbol{C} = \eta \boldsymbol{v}, \tag{16}
$$

with η the viscosity of the fluid. This way, we obtain a full 3D generalization of Newton's second law per unit volume, describing not only a longitudinal force density field $\boldsymbol{L}$ but also a a rotational or angular force density field $\boldsymbol{R}$:

$$
\rho \frac{d\boldsymbol{v}}{dt} = -\nabla^2 \boldsymbol{C} = -\nabla^2 \eta \boldsymbol{v} = -(\boldsymbol{L} + \boldsymbol{R}). \tag{17}
$$

$$
\rho \frac{d\boldsymbol{v}}{dt} = -\text{LaPlace}(\boldsymbol{C}) = -\text{LaPlace}(\eta \boldsymbol{v}) = -(\boldsymbol{L} + \boldsymbol{R}).
$$

When we divide this by mass density $\rho,$ we obtain the momentum diffusion equation:

$$
\boldsymbol{a} = \frac{d\boldsymbol{v}}{dt} = -\nu \nabla^2 \boldsymbol{v}, \tag{18}
$$

$$\boldsymbol{a}=\frac{d\boldsymbol{v}}{dt}=-\nu\,\mathrm{LaPlace}(\boldsymbol{v}),$$

with $\boldsymbol{a}$ the acceleration field in [m/s^2] and ν the momentum diffusivity or kinematic viscosity, defined by:

$$\nu=\frac{\eta}{\rho} \tag{19}$$

From here, we can also define a second order momentum diffusion equation:

$$\boldsymbol{j}=\frac{d\boldsymbol{a}}{dt}=-\nu\nabla^2\boldsymbol{a}=-\nu\nabla^2(-\nu\nabla^2\boldsymbol{v})=\nu^2\nabla^4\boldsymbol{v}, \tag{20}$$

$$\boldsymbol{j}=\frac{d\boldsymbol{a}}{dt}=-\nu\,\mathrm{LaPlace}(\boldsymbol{a})=-\nu\,\mathrm{LaPlace}(-\nu\,\mathrm{LaPlace}(\boldsymbol{v}))=\nu^2\,\mathrm{LaPlace}(\mathrm{LaPlace}(\boldsymbol{v})),$$

which we can work out further to define the intensity field $\boldsymbol{I}$ in Watts per square meter [W/m^2], representing a heat flux density in case of the aether:

$$\rho\frac{d\boldsymbol{a}}{dt}=\rho\frac{d^2\boldsymbol{v}}{dt^2}=-\nabla^2\boldsymbol{I}=-\nabla^2\eta\,\boldsymbol{a}=-\nu\nabla^2(\boldsymbol{L}+\boldsymbol{R})=-\nu\nabla^2(\nabla^2\eta\boldsymbol{v})=-\eta\,\nu\nabla^4\boldsymbol{v} \tag{21}$$

$$\rho\frac{d\boldsymbol{a}}{dt}=\rho\frac{d^2\boldsymbol{v}}{dt^2}=-\mathrm{LaPlace}(\boldsymbol{I})=-\mathrm{LaPlace}(\eta\,\boldsymbol{a})=-\nu\,\mathrm{LaPlace}(\boldsymbol{L}+\boldsymbol{R})=-\nu\,\mathrm{LaPlace}(\mathrm{LaPlace}(\eta\,\boldsymbol{v}))$$

-:-

Some quick additions and notes between -:- markings

A table with the different units of measurement for the different fields defined:

	M = ρ **v**	**X** = λ **v**	**C** = η **v** = $\nu\rho\boldsymbol{v}$	**I** = κ **v** = η d**v**/dt = $\nu\rho\boldsymbol{a}$ = ν(**L**+**R**)
C, I	[kg/m^2-s]	[kg/s], [N-s/m]	[kg/s^2], [N/m] [J/m^2], [Pa-m] (Ampère)	[kg/s^3], [N/m-s], [J/m^2-s], [Pa-m/s], [W/m^2] (radiosity $\mathbf{J}_e$, intensity **I**, heat flux density)
P, **Ω** T, **ω**	[kg/m^3-s] (dρ/dt)	[kg/m-s], [N-s/m^2], [J-s/m^3], [Pa-s]	[kg/m-s^2], [N/m^2], [J/m^3], [Pa], (momentum flux, energy density)	[kg/m-s^3], [N/m^2-s], [J/m^3-s], [Pa/s], [W/m^3], [K] (temperature)
L, **R**	[kg/m^4-s]	[kg/m^2-s], [N-s/m^3], [Pa-s/m] (ρ **v**)	[kg/m^2-s^2], [N/m^3], [Pa/m] (ρ **a**)	[kg/m^2-s^3], [N/m^3-s], [J/m^4-s], [Pa/m-s], [J/m^4-s], [W/m^4] (ρ **j**)
J = curl **R**			[kg/m^3-s^2], [N/m^4], [Pa/m^2] (d^{2}ρ/dt^2)	
ρ_q			[kg/m^3-s] (dρ / dt)	
q			[kg/s]	

Important detail is that we start at a viscous fluid, or at least a substance whereby momentum diffusion takes place. The interesting things is that intensity $\boldsymbol{I}$ yields units of measurement which describe elastic behavior (most right column), while this is also the time derivative of viscous behavior in the column to the left ($\boldsymbol{C}$).

What all of this is, is Newton's second law in terms of densities (equation (17)) and working this out consequently.

This way, we obtain very concrete fields, of which it is exactly clear what they mean and represent.

Latest find is that I now also have a proper definition for the electric field:

$$\boldsymbol{E} = \frac{1}{\rho_q} \boldsymbol{L}, \tag{22}$$

with $\boldsymbol{L}$ as defined in equation (14) en ρ_q the (local) charge density.

Coulomb's law then becomes:

$$\boldsymbol{F} = q\,\boldsymbol{E} = \frac{q}{\rho_q} \boldsymbol{L} \tag{23}$$

And the electric (scalar) potential φ can subsequently be defined as:

$$\varphi = \frac{1}{\rho_q} P, \tag{24}$$

with P the scalar pressure in Pascal [Pa], yielding a unit of measurement in meters squared per second [m^2/s] for the scalar electric potential, which happens to be the same unit of measurement as the kinematic viscosity ν .

Furthermore, what's really interesting is that with the “momentum diffusion equation” we have found a relation between the *time* derivative of velocity and the second order *spatial* derivative of velocity times nu, a constant (equation (18)):

a = dv/dt = - LaPlace (nu v)   (= - d^2(nu v)/dx^2 in 1D),

From this, it seems like we can actually define the time derivative operation itself:

d/dt = - nu LaPlace()   (= - nu d^2/dx^2 in 1D),

which seems quite profound.

-:-

This definition allows us to work with the vector wave equation, which has three independent solutions[37], the vector spherical harmonics:

$$\nabla^2 \boldsymbol{F} + k^2 \boldsymbol{F} = \nabla \nabla \cdot \boldsymbol{F} - \nabla \times \nabla \times \boldsymbol{F} + k^2 \boldsymbol{F} = 0. \tag{25}$$

$$\text{LaPlace}(\boldsymbol{F}) + k^2 \boldsymbol{F} = grad \, \text{div} \, \boldsymbol{F} - curl \, curl \, \boldsymbol{F} + k^2 \boldsymbol{F} = 0.$$

-:-

And here we have the problem with quantum mechanics, whereby they work with a complex wave **function**. In that case, you only have two axis, the real and the imaginary, which is simply not sufficient to describe phenomena in three dimensions.

This one is a full 3D vector equation, which **is** sufficient to describe phenomena in three dimensions….
-:-

Now let us consider the Cauchy momentum equation without external forces working on the fluid:

$$\rho \frac{d \boldsymbol{v}}{dt} = -\nabla \cdot \boldsymbol{P}, \tag{26}$$

$$\rho \frac{d \boldsymbol{v}}{dt} = -grad \, \boldsymbol{P},$$

with $\boldsymbol{P}$ the Cauchy stress tensor, which also has a unit of measurement in [N/m^2] or [Pa] and is a central concept in the linear theory of elasticity for continuum solid bodies in static equilibrium, when the resultant force and moment on each axis is equal to zero. It can be demonstrated that the components of the Cauchy stress tensor in every material point in a body satisfy the equilibrium equations and according to the principle of conservation of angular momentum, equilibrium requires that the summation of moments with respect to an arbitrary point is zero, which leads to the conclusion that the stress tensor is symmetric, thus having only six independent stress components, instead of nine.

In our model, we have seven independent stress components, namely the scalar and two vectorial solutions, the solutions from the vector wave equation.

From this momentum equation, the Navier-Stokes equations can be derived, of which the most general one without external (gravitational) forces is:

$$\rho \frac{D \boldsymbol{v}}{Dt} = \rho \left(\frac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{v} \cdot \nabla \boldsymbol{v} \right) = -\nabla p + \nabla \cdot \left\{ \eta \left(\nabla \boldsymbol{v} + (\nabla \boldsymbol{v})^T - \frac{2}{3} (\nabla \cdot \boldsymbol{v}) \boldsymbol{I} \right) + \zeta (\nabla \cdot \boldsymbol{v}) \boldsymbol{I} \right\}, \tag{27}$$

with p the pressure, **I** the identity tensor and ζ the volume, bulk or second viscosity. This can be re-written to:

$$\rho \frac{\delta v}{\delta t} = -\nabla p - \rho (v \cdot \nabla v) + \eta \nabla \cdot \left(\nabla v + (\nabla v)^T \right) + \left(\zeta - \frac{2\eta}{3} \right) (\nabla \cdot \nabla \cdot v) I,$$

This is also a second order equation, whereby notably for the viscous term $\eta \nabla \cdot \left(\nabla v + (\nabla v)^T \right)$ the order of the differential operators is reversed compared to the definition of the second spatial derivative, the vector Laplace operator, while for the elastic term, $\left(\zeta - \frac{2\eta}{3} \right) (\nabla \cdot \nabla \cdot v) I$, the divergence of the divergence is taken. Also, a separate term is introduced for pressure as well as a convective term, $\rho (v \cdot \nabla v)$. All this not only causes the complexity of the equations to increase dramatically while introducing redundancy in the symmetric stress tensor, it also ignores the fundamental symmetry between the compressible, irrotational components and the incompressible, solenoidal components as prescribed by the Helmholtz decomposition.

When we compare this with our proposal, we end up with two fundamentally different approaches:

1. A solution that has only viscosity and yields harmonic solutions c./q. builds upon deterministic harmonics;
2. A solution that has both viscosity as well as elasticity, which builds upon statistical mechanics and thus requires randomness and is therefore non-deterministic.

Obviously, the units of measurement used within the fluid dynamics domain are not the same as those used within Maxwell's equations, which essentially describe a phenomenological model that is based upon the assumption that some kind of fundamental quantity called "charge" exists, to which a unit of measurement in Coulombs [C] has been assigned. All of the units of measurement within the electromagnetic domain can be derived from the Coulomb, so if we can define what charge actually is, we can connect the electromagnetic domain with the fluid dynamics domain and integrate these two domains in order to come to an integrated "theory of everything" that is based on a single hypothesis:

The medium wherein electromagnetic phenomena propagate behaves like a fluid/gas and should therefore be described as such.

Stowe and Mingst proposed[38] (eq. 25) that the charge to mass ratio of a "charged" particle gives its characteristic oscillation frequency *f*, which is thus defined as:

$$f = q / m$$

(eq. 16)

This frequency can also be used to obtain a relation between temperature and the characteristic oscillation frequency of a charged particle, which accordingly works out to 2.8 K for the electron, which suggests a link between the characteristic oscillation frequency of the electron and the observed Cosmic Microwave Background.

From this equation, we can work out a unit of measurement for charge, which results in a unit of measurement for charge *q* in kilograms per second [kg/s], and we can define the Coulomb as:

$$1 \text{ Coulomb} = 1 \text{ kilogram per second}$$

(eq. 17)

-:-

The units of measurement and definitions below here will have to be re-evaluated and re-considered. The pieces of the puzzle fit together better and better, but I'm not there yet. In other words: the rest of this section will pretty much have to be re-written.

Derivation of the unit of measurement for charge goes as follows and does not rely on Stowe's proposal I used just above:

Ampere's law with Maxwell's extensions (Faraday's law) is given by:

curl H = J + epsilon_0 dE/dt.

As argued, Faraday's law is incorrect, and thus we should write:

J = curl H.

Equate H to R, the rotational force density field we defined, and we obtain:

J = curl R,

as I wrote in the table.

Then, we obtain a unit of measurement for J, current density, in [kg/mˆ3-s^2] or [A/mˆ3] and we can define the Ampere as:

1 Ampere = 1 kilogram per second squared [kg/sˆ2],

And since 1 Ampere equals 1 Coulomb per second, we can define the Coulomb as:

1 Coulomb = 1 kilogram per second [kg/s].

-:-

Now let us consider the original form of Ampère's circuital law in Maxwell's equations in macroscopic formulation:

$$\mathbf{J} = \nabla \times \mathbf{H}_{em}$$

(eq. 18)

And consider our definition for the vorticity field $\boldsymbol{\omega}$:

$$\boldsymbol{\omega} = \nabla \times \mathbf{B}_{fd} = \nabla \times (\nabla \times \mathbf{A}_{fd})$$

(eq. 19)

We can connect these two domains by redefining the current density **J** and Ampère's circuital law to:

$$\mathbf{J} = e\,\boldsymbol{\omega} = e\,\nabla\times\mathbf{H}_{em},$$

(eq. 20)

with *e* the value for elemental charge, approximately 1.602176634×10−19 C or kg/s, and we come to the realization that the quantity we call "current density" actually represents the vorticity of the medium and that the electromagnetic domain can be fully and correctly described by a fluid dynamics model, provided we adhere to the fundamental theorem of vector calculus to define all of our fields within both domains. Therefore, we redefine the magnetic vector potential $\mathbf{A}_{em}$ to:

$$\mathbf{H}_{em} = \nabla\times\mathbf{A}_{em}$$

(eq. 21)

Since both domains are now fully interchangeable, and all fields are uniquely defined as solutions of the vector Laplace equation, we can establish that with deriving all fields from this equation, we have eliminated "gauge freedom" and since we know these equations can be transformed using the Galilean coordinate transform, we have also eliminated the need for the Lorentz transform.

With this application of the fundamental theorem of vector calculus, we have thus come to a revised version of the Maxwell equations that not only promises to resolve all of the problems that have been found over the years, we also obtain a model that is easy to interpret and can be easily simulated and visualized with finite-difference time-domain methods (FTDT) as well.

Now let us consider the difference between the definition we found for **E** and the corresponding definition in Maxwell's equations:

$$\mathbf{E}_{em} = -\nabla\Phi_{em} - \partial\mathbf{A}_{em}/\partial t$$

(eq. 22)

When considered from the presented perspective, this is what breaks the fundamental result of Helmholtz' decomposition, namely the decomposition into a rotation free translational component and a divergence free rotational component, since $\mathbf{A}_{em}$ is not rotation free and therefore neither is its time derivative.

When taking the curl on both sides of this equation, we obtain the Maxwell-Faraday equation, representing Faraday's law of induction:

$$\nabla\times\mathbf{E}_{em} = -\partial\mathbf{B}_{em}/\partial t$$

(eq. 23)

Faraday's law of induction is a basic law of electromagnetism predicting how a magnetic field will interact with an electric circuit to produce an electromotive force (EMF), which is thus a law that applies at the macroscopic level. It is clear that this law should not be entangled with a model for the medium and therefore our revision should be preferred.

We can now also work out units of measurements for the electromagnetic domain in terms of the fluid dynamics domain, since both domains are now interchangeable. As shown, the Coulomb has a unit of measurement in [kg/s] within the fluid dynamics domain and from here, we can work out all associated units of measurement for the electromagnetic domain.

Let us start with permittivity, denoted by ε. This has a SI unit of measurement in $[C^2\ m^{-2}\ N^{-1}]$, whereby the Newton is defined in $[kg\ m\ s^{-2}]$. When we substitute the Newton and the Coulomb in this definition, we obtain a unit of measurement in $[kg/m^3]$, which thus denotes mass density, commonly denoted by *ρ*, and defined as mass divided by volume. In other words: permittivity is now one and the same thing as the mass density of the medium.

The SI unit of measurement for the electric field is in Newtons per Coulomb [N/C], or Volts per meter [V/m]. By substituting the Newton and the Coulomb in this definition, we obtain a unit of measurement in [m/s] for the **E** field and $[m^2/s]$ for the electric potential Φ_{fd} in Volts [V], which denotes the kinematic viscosity or momentum diffusivity of the medium, denoted by *ν*, and defined as the ratio of the dynamic or absolute viscosity μ to the mass density of the medium ρ:

$$\nu = \mu / \rho = \mu / \varepsilon$$

(eq. 24)

and we can define:

$$1 \text{ Volt} = \mu/\varepsilon \text{ square meters per second.}$$

(eq. 25)

Since the unit of measurement for $\mathbf{E}_{em}$ and $\mathbf{H}_{em}$ must both be the same and thus have a unit of measurement in [m/s], we can use substitution in our redefined Ampère's circuital law and thus obtain a unit of measurement in $[kg/s^2]$ for the Ampère. If we use the definition for the Ampère in Coulombs per second, we also obtain a unit of measurement in $[kg/s^2]$. And when we derive the definition for the Ampère from our definition for current density:

$$\mathbf{J} = e\ \boldsymbol{\omega},$$

(eq. 26)

we also obtain a unit of measurement in Coulombs per second [C/s] or $[kg/s^2]$, and we can define the Ampère as follows:

$$1 \text{ Ampère} = e \text{ kilograms per square second,}$$

(eq. 27)

with *e* the value for elemental charge.

The magnetic field $\mathbf{B}_{em}$ has a unit of measurement in Tesla [T], which is defined in SI units as $[kg/s^2\text{-}A]$. If we were to substitute the Ampère in this definition, $\mathbf{B}_{em}$ would become dimensionless, so we have to derive the unit of measurement for $\mathbf{B}_{em}$ from the definition:

$$\mathbf{B}_{em} = \mu\ \mathbf{H}_{em}$$

(eq. 28)

This can be accomplished by equating magnetic permeability, denoted by μ, in Henrys per meter [H/m], to the viscosity of the medium, also denoted by μ, in Newton seconds per square meter [N s m^{-2}], Pascal seconds [Pa s] or kilograms per meter per second [kg m^{-1} s^{-1}] in base SI units, which results in the unit of measurement for magnetic permeability to become also expressible in Coulombs per meter [C/m]. This results in a unit of measurement for the magnetic field $\mathbf{B}_{em}$ in [kg/s^2] or Ampère, and we can define:

$$1 \text{ Tesla} = 1 \text{ Ampère} \qquad \text{(eq. 29)}$$

This way, all of the units of measurement within the electromagnetic domain can be derived, which is left as an exercise to the reader.

Discussion and Conclusions

We have shown that the terms in the Laplace operator can be written out to define a complete and mathematically consistent whole of four closely related vector fields which by definition form solutions to the vector Laplace equation, a result that has tremendous consequences for both the analytical analysis of the electromagnetic field as well as fluid dynamics vector theory, such as weather forecasting, oceanography and mechanical engineering. The symmetry between the fields thus defined is fundamental and has been mathematically proven to be correct, so it is vital to maintain this fundamental symmetry in our physics equations.

Revising Maxwell equations by deriving directly from a superfluid medium model using the Laplace operator, we have called upon fluid dynamics vector theory for an ideal, compressible, non-viscous Newtonian fluid that has led to equations which are known to be mathematically consistent, are known to be free of singularities and are invariant to the Galilean transform as well. This results in an integrated model which has only three fundamental units of measurement: mass, length and time and also explains what “charge” is: a compression/decompression oscillation of “charged” particles.

As is known from fluid dynamics, these revised Maxwell equations predict three types of wave phenomena, which we can easily relate to the observed phenomena:

1 Longitudinal pressure waves, Tesla’s superluminal waves[39] c.q. the super luminal longitudinal dielectric mode, which he found to propagate at a speed of 471,240 kilometers per second, within 0.1% of $\pi/2$ times the speed of light. The factor $\pi/2$ coincides with the situation whereby the theoretical reactance of a shorted lossless transmission line goes to infinity[40] (eq 1.2) and thus does not support an electromagnetic wave propagation mode;

2 “Transverse” “water” surface waves, occurring at the boundary of two media with different densities such as the metal surfaces of an antenna and air, aka the “near field”;

3 Vortices and/or vortex rings, the “far field”, which is known to be quantized and to incorporate a thus far mysterious mixture of “particle” and “wave” properties, the so called “wave particle duality” principle.

Even though the actual wave equations for these three wave types still need to be derived, we can already conclude these to exist and predict a number of their characteristics, because of the integration of the electromagnetic domain with the fluid dynamics domain. The latter has a tremendous advantage, namely that dynamic phenomena known to occur in fluids and gasses can be considered to also occur in the medium.

Further Research

Theoretical

While the revised Maxwell equations presented in this paper describe the motions of the medium accurately in principle, the actual wave equations for the three predicted wave types still need to be derived and worked out. This is particularly complicated for the “transverse” “water” surface wave, because of the fact that in current fluid dynamics theory the potential fields have not been defined along the Helmholtz decomposition defined by the vector Laplacian as we proposed, which leads to non-uniquely defined fields and associated problems with boundary conditions. In order to derive a wave equation for the “transverse” surface wave, the incompressibility constraint would have to be removed from the Saint-Venant equations[41] and these would subsequently need to be fully worked out using vector calculus methods.

Furthermore, we have also argued that Faraday's law should not be entangled with the model for the medium, which leaves us without revised equations for Faraday's law of induction. This leads to the question of why a DC current trough a wire loop results in a magnetic field, but the magnetic field of a permanent magnet does not induce a current in a wire wound around it. A similar question arises when a (neodymium) magnet is used as an electrode in an electrolysis experiment, which results in a vortex becoming visible in the electrolyte above the magnet.

It is expected the answers to these questions as well as Faraday’s law of induction can be worked out by considering the physics of the irrotational vortex, given that we found that the current density is actually one and the same thing as the vorticity of the medium, apart from a constant. In the absence of external forces, a vortex evolves fairly quickly toward the irrotational flow pattern, where the flow velocity $\mathbf{v}$ is inversely proportional to the distance r. The fluid motion in a vortex creates a dynamic pressure that is lowest in the core region, closest to the axis, and increases as one moves away from it. It is the gradient of this pressure that forces the fluid to follow a curved path around the axis and it is this pressure gradient that is directly related to the velocity potential Φ_{fd} c.q. the velocity field component $\mathbf{E}_{fd}$.

Practical

The revised Maxwell equations presented in this paper open the possibilities of further considerations and research into the properties of the dielectric and gravitational fields and associated wave phenomena. Because both of these fields are considered as one and the same within the above presented revised Maxwell paradigm, a wide range of possible applications become conceivable, some of which are hardly imaginable from within the current paradigm and/or are highly speculative while others are more straightforward.

Superluminal communication
This is the most direct application of the theory presented in this paper, which is supported by a number of sources mentioned in the abstract, the oldest of which dates back to 1834, some theoretical methods[42,43,44,45] as well as some preliminary experimental work by the author. There is active and ongoing experimental research in this area.

Experiments regarding gravitational effects, such as aimed at obtaining thrust.

The Biefeld-Brown effect is an electrical phenomenon that has been the subject of extensive research involving charging an asymmetric capacitor to high voltages and the effect is commonly attributed to corona discharges which occur only at the sharp electrode, which causes an imbalance in the number of positive and negative ions created in comparison to when a symmetric capacitor is used.

However, according to a report[46] by researchers from the Army Research Laboratory (ARL), the effects of ion wind was at least three orders of magnitude too small to account for the observed force on the asymmetric capacitor in the air. Instead, they proposed that the Biefeld–Brown effect may be better explained using ion drift instead of ion wind. This was later confirmed by researchers from the Technical University of Liberec[47].

If this is correct, then the need for an asymmetric capacitor raises the question if the resulting diverging electric field can indeed be used to obtain thrust by working on an electrically neutral dielectric, in this case a dielectric consisting of air and net neutral ions, and how this results in a net force acting upon the capacitor plates. It is known that a dielectric is always drawn from a region of weak field toward a region of stronger field. It can be shown that for small objects the force is proportional to the gradient of the square of the electric field, because the induced polarization charges are proportional to the fields and for given charges the forces are proportional to the field as well. There will be a net force only if the square of the field is changing from point to point, so the force is proportional to the gradient of the square of the field[48].

Another line of research in this regard has to do with the gravitational force itself, which in our model is proposed to be caused by longitudinal dielectric flux, which causes a pushing and not a pulling force. This is supported by Van Flandern[49], who determined that with a purely central pulling force and a finite speed of gravity, the forces in a two-body system no longer point toward the center of mass, which would make orbits unstable. The fact alone that a central pulling gravity force requires a practically infinite speed makes clear that pulling gravity models are untenable and recourse must be taken to a Lesagian type of pushing gravity model. The longitudinal dielectric flux which would describe gravity in our model is probably caused by cosmic (microwave) background radiation. If this naturally occurring flux had an arbitrary frequency spectrum, superconductors would reflect this flux and would thus shield gravity, which does not happen.

However, acceleration fields outside a rotating superconductor were found[50,51], which are referred to as Gravitomagnetic effects, and also anomalous acceleration signals, anomalous gyroscope signals and Cooper pair mass excess were found in experiments with rotating superconductors[52].

It can be speculated that the relation Stowe and Mingst found between the characteristic oscillation frequency of the electron and the cosmic microwave background radiation is what causes the spectrum of the gravitational flux and that this is related to the characteristic oscillation frequencies of the electron, neutron and proton as well. If that is the case, then the incoming flux would resonate with the oscillating particles within the material at these specific frequencies, which would therefore not be blocked/reflected but would be absorbed/re-emitted along Huygens' principle.

It can further be speculated that when objects are rotated, their “clock”, the characteristic oscillation frequency of the elemental particles making up the material, would be influenced, causing them to deviate from the specific frequencies they otherwise operate at. It is conceivable that this would result in a condition whereby superconductors would indeed reflect the naturally occurring gravitational flux, which could explain this anomaly.

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This work would not have been possible without the groundbreaking work of Paul Stowe and Barry Mingst, who succeeded in integrating the gravitational domain with the electromagnetic domain within a single superfluid based model. It is this integration that resolves the classic problems associated with aether based theories, namely that because the gravitational field was considered to be separate from the electromagnetic domain, the movements of planetary bodies would necessarily result in measurable disturbances in the medium. When no such disturbances were found in the Michelson-Morley experiment, the aether hypothesis was considered as having been disproven. But because the gravitational force is now considered to be a force caused by longitudinal dielectric waves, which propagate trough the medium, this argument no longer applies. And therewith there is no longer any reason to disregard an aether based theory as a basis for theoretical physics.

This work would also not have been possible without the work of Eric Dollard, N6KPH, who replicated a lot of Tesla's experiments in the 1980's. It is his demonstrations and analysis of Tesla's work that enabled the very consideration of an aether based theory as an alternative to the current theoretical model.

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